Qualifying Exam Syllabus

Esme Bajo

May 27, 2021, 10am - 1pm

Committee: Fraydoun Rezakhanlou (Exam Chair), Matthias Beck (Co-advisor, SFSU), Sylvie Corteel (Co-advisor), Nikhil Srivastava.

1 Major topic: Algebraic Combinatorics (Algebra)

References: Sagan, *Combinatorics: The Art of Counting*, Chapters 1-5, 7-8; Beck and Sanyal, *Combinatorial Reciprocity Theorems*, Chapters 2, 4.

- Enumerative Combinatorics: permutations, trees, partitions, lattice paths, reflection principle, inclusion-exclusion, sign-reversing involutions, Lindström-Gessel-Viennot lemma, Matrix Tree Theorem.

- Generating Functions: ordinary generating functions, reciprocity, compositions, plane partitions, exponential generating functions, q-analogs, labeled structures, Exponential Formula, Lagrange Inversion Theorem.

- Partially Ordered Sets: order polynomials, order ideals, Möbius functions, Möbius inversion, zeta polynomials, Eulerian posets.

- Symmetric Functions: Young Tableaux, Schur basis, hooklengths, P-partitions, RSK correspondence, chromatic symmetric functions, reverse P-partitions.

2 Major topic: Discrete Geometry (Algebra)

References: Beck and Sanyal, *Combinatorial Reciprocity Theorems*, Chapters 3, 5-7; Beck and Robbins, *Computing the Continuous Discretely*, Chapters 2-4, 9.

- Polytopes: Polytopes, polyhedra, cones, operations on polytopes, main theorem for polytopes, faces, Dehn-Sommerville equations, face lattices, simplicial polytopes, subdivisions and triangulations, half-open decompositions.

- Geometric Combinatorics: Ehrhart polynomials, Ehrhart quasipolynomials, Ehrhart series, integer-point transforms, Hilbert series of cones, Ehrhart-Macdonald reciprocity, Stanley reciprocity for cones, Brion's theorem, Euler characteristics, flats and regions of hyperplane arrangements.

- Special Examples: zonotopes, reflexive polytopes, order polytopes, chain polytopes.

3 Minor topic: Complex Analysis (Analysis)

References: Stein and Shakarchi, *Complex Analysis*, Chapters 1-3, 8.

- Holomorphic Functions: Cauchy-Riemann equations, Taylor and Laurent series, Liouville's theorem, argument principle, Rouché's theorem, open mapping theorem, maximum modulus principle.

- Complex Integration: Cauchy integral formula, Morera's theorem, residue theorem.

- Conformal Mappings: fractional linear transformations, automorphisms of the disc, Schwarz lemma, Riemann mapping theorem.